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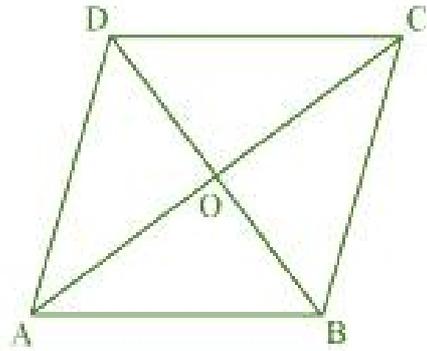
Miscellaneous Exercise on Chapter 12

Question 1:

Three vertices of a parallelogram ABCD are A (3, -1, 2), B (1, 2, -4) and C (-1, 1, 2). Find the coordinates of the fourth vertex.

Answer 1:

The three vertices of a parallelogram ABCD are given as A (3, -1, 2), B (1, 2, -4), and C (-1, 1, 2). Let the coordinates of the fourth vertex be D (x, y, z).



We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, AC and BD bisect each other.

∴ Mid-point of AC = Mid-point of BD

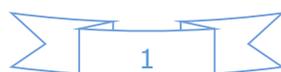
$$\Rightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2} \right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$

$$\Rightarrow (1, 0, 2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2} \right)$$

$$\Rightarrow \frac{x+1}{2} = 1, \frac{y+2}{2} = 0, \text{ and } \frac{z-4}{2} = 2$$

$$\Rightarrow x = 1, y = -2, \text{ and } z = 8$$

Thus, the coordinates of the fourth vertex are (1, -2, 8).





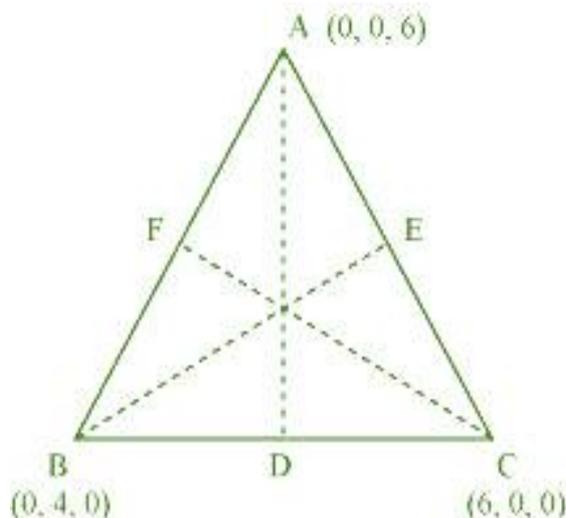
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Question 2:

Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

Answer 2:

Let AD, BE, and CF be the medians of the given triangle ABC.



Since AD is the median, D is the mid-point of BC.

$$\therefore \text{Coordinates of point D} = \left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2} \right) = (3, 2, 0)$$

$$AD = \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$$

Since BE is the median, E is the mid-point of AC.

$$\therefore \text{Coordinates of point E} = \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2} \right) = (3, 0, 3)$$

$$BE = \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2} = \sqrt{9+16+9} = \sqrt{34}$$

Since CF is the median, F is the mid-point of AB.

$$\therefore \text{Coordinates of point F} = \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2} \right) = (0, 2, 3)$$

$$\text{Length of CF} = \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$$

Thus, the lengths of the medians of ΔABC are $7, \sqrt{34}$, and 7 .

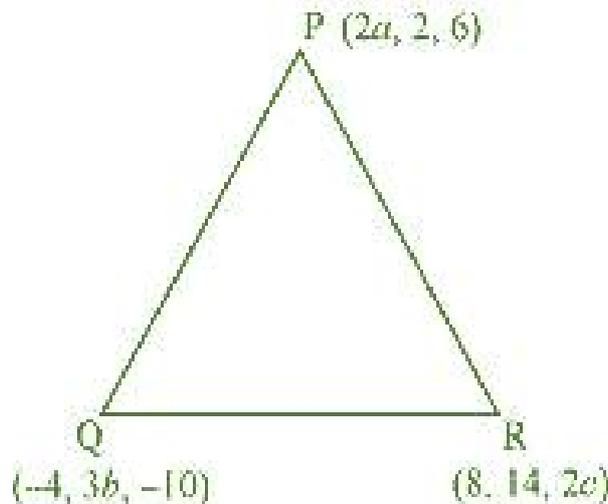
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Question 3:

If the origin is the centroid of the triangle PQR with vertices P $(2a, 2, 6)$, Q $(-4, 3b, -10)$ and R $(8, 14, 2c)$, then find the values of a , b and c .

Answer 3:



It is known that the coordinates of the centroid of the triangle, whose vertices are $(x_1,$

$y_1, z_1)$, (x_2, y_2, z_2) and (x_3, y_3, z_3) , are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.

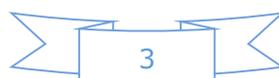
Therefore, coordinates of the centroid of ΔPQR

$$= \left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

It is given that origin is the centroid of ΔPQR .

$$\begin{aligned}\therefore (0, 0, 0) &= \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right) \\ \Rightarrow \frac{2a+4}{3} &= 0, \frac{3b+16}{3} = 0 \text{ and } \frac{2c-4}{3} = 0 \\ \Rightarrow a &= -2, b = -\frac{16}{3} \text{ and } c = 2\end{aligned}$$

Thus, the respective values of a , b , and c are $-2, -\frac{16}{3}$, and 2 .





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Question 4:

Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point P (3, -2, 5).

Answer 4:

If a point is on the y-axis, then x-coordinate and the z-coordinate of the point are zero. Let A (0, b, 0) be the point on the y-axis at a distance of $5\sqrt{2}$ from point P (3, -2, 5).

Accordingly, $AP = 5\sqrt{2}$

$$\therefore AP^2 = 50$$

$$\Rightarrow (3-0)^2 + (-2-b)^2 + (5-0)^2 = 50$$

$$\Rightarrow 9 + 4 + b^2 + 4b + 25 = 50$$

$$\Rightarrow b^2 + 4b - 12 = 0$$

$$\Rightarrow b^2 + 6b - 2b - 12 = 0$$

$$\Rightarrow (b+6)(b-2) = 0$$

$$\Rightarrow b = -6 \text{ or } 2$$

Thus, the coordinates of the required points are (0, 2, 0) and (0, -6, 0).

Question 5:

A point R with x-coordinate 4 lies on the line segment joining the points P (2, -3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

[Hint suppose R divides PQ in the ratio k: 1. The coordinates of the point R are given by

$$\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right)]$$

Answer 5:

The coordinates of points P and Q are given as P (2, -3, 4) and Q (8, 0, 10).

Let R divide line segment PQ in the ratio k:1.

Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1} \right) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1} \right)$$

It is given that the x-coordinate of point R is 4.





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$$\therefore \frac{8k+2}{k+1} = 4$$

$$\Rightarrow 8k+2 = 4k+4$$

$$\Rightarrow 4k = 2$$

$$\Rightarrow k = \frac{1}{2}$$

Therefore, the coordinates of point R are

$$\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1} \right) = (4, -2, 6)$$

Question 6:

If A and B be the points (3, 4, 5) and (-1, 3, -7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Answer 6:

The coordinates of points A and B are given as (3, 4, 5) and (-1, 3, -7) respectively. Let the coordinates of point P be (x, y, z).

On using distance formula, we obtain

$$\begin{aligned} PA^2 &= (x-3)^2 + (y-4)^2 + (z-5)^2 \\ &= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z \\ &= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50 \end{aligned}$$

$$\begin{aligned} PB^2 &= (x+1)^2 + (y-3)^2 + (z+7)^2 \\ &= x^2 + 2x + y^2 - 6y + z^2 + 14z + 59 \end{aligned}$$

Now, if $PA^2 + PB^2 = k^2$, then

$$\begin{aligned} (x^2 - 6x + y^2 - 8y + z^2 - 10z + 50) + (x^2 + 2x + y^2 - 6y + z^2 + 14z + 59) &= k^2 \\ \Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 &= k^2 \\ \Rightarrow 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) &= k^2 - 109 \\ \Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z &= \frac{k^2 - 109}{2} \end{aligned}$$

Thus, the required equation is $x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$