(Chapter – 12)

Question 1:

Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1) (ii) (−3, 7, 2) and (2, 4, −1)

(iii) (−1, 3, −4) and (1, −3, 4) (iv) (2, −1, 3) and (−2, 1, 3)

Answer 1:

The distance between points P(x_1, y_1, z_1) and P(x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(i) Distance between points (2, 3, 5) and (4, 3, 1)

$$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$$
$$= \sqrt{(2)^2 + (0)^2 + (-4)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

(ii) Distance between points (−3, 7, 2) and (2, 4, −1)

$$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$$
$$= \sqrt{(5)^2 + (-3)^2 + (-3)^2}$$
$$= \sqrt{25+9+9}$$
$$= \sqrt{43}$$

(iii) Distance between points (−1, 3, −4) and (1, −3, 4)

$$= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2}$$
$$= \sqrt{(2)^2 + (-6)^3 + (8)^2}$$
$$= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26}$$

(iv) Distance between points (2, −1, 3) and (−2, 1, 3)

$$= \sqrt{(-2-2)^2+(1+1)^2+(3-3)^2}$$
$$= \sqrt{(-4)^2+(2)^2+(0)^2}$$
$$= \sqrt{16+4}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

Question 2:

Show that the points (−2, 3, 5), (1, 2, 3) and (7, 0, −1) are collinear.

Answer 2:

Let points (−2, 3, 5), (1, 2, 3), and (7, 0, −1) be denoted by P, Q, and R respectively.
Points P, Q, and R are collinear if they lie on a line.

$$PQ = \sqrt{(1+2)^2+(2-3)^2+(3-5)^2}$$
$$= \sqrt{(3)^2+(-1)^2+(-2)^2}$$
$$= \sqrt{9+1+4}$$
$$= \sqrt{14}$$

$$QR = \sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}$$
$$= \sqrt{(6)^2+(-2)^2+(-4)^2}$$
$$= \sqrt{36+4+16}$$
$$= \sqrt{56}$$
$$= 2\sqrt{14}$$

$$PR = \sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}$$
$$= \sqrt{(9)^2+(-3)^2+(-6)^2}$$
$$= \sqrt{81+9+36}$$
$$= \sqrt{126}$$
$$= 3\sqrt{14}$$

Here, PQ + QR $= \sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$ = PR

Hence, points P(−2, 3, 5), Q(1, 2, 3), and R(7, 0, −1) are collinear.

Question 3:

Verify the following:

(i) (0, 7, −10), (1, 6, −6) and (4, 9, −6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (−1, 6, 6) and (−4, 9, 6) are the vertices of a right angled triangle.

(iii) (−1, 2, 1), (1, −2, 5), (4, −7, 8) and (2, −3, 4) are the vertices of a parallelogram.

Answer 3:

(i) Let points (0, 7, −10), (1, 6, −6), and (4, 9, −6) be denoted by A, B, and C respectively.

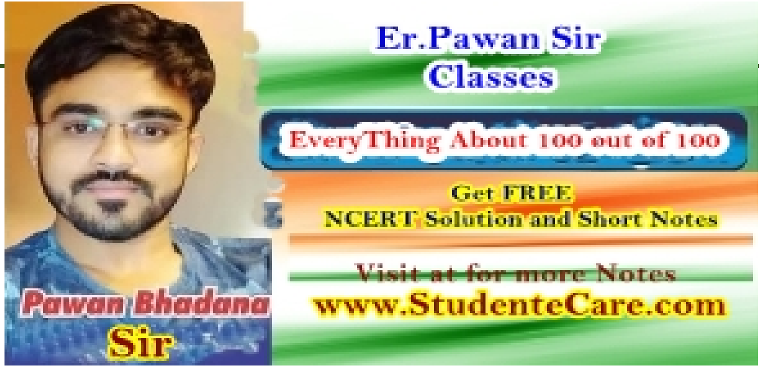
$$\begin{aligned} AB &= \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} \\ &= \sqrt{(1)^2 + (-1)^2 + (4)^2} \\ &= \sqrt{1+1+16} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} \\ &= \sqrt{(3)^2 + (3)^2} \\ &= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} CA &= \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2} \\ &= \sqrt{(-4)^2 + (-2)^2 + (-4)^2} \\ &= \sqrt{16+4+16} = \sqrt{36} = 6 \end{aligned}$$

Here, AB = BC ≠ CA

Thus, the given points are the vertices of an isosceles triangle.



(ii) Let (0, 7, 10), (–1, 6, 6), and (–4, 9, 6) be denoted by A, B, and C respectively.

$$
\begin{aligned}
AB &= \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2} \\
&= \sqrt{(-1)^2+(-1)^2+(-4)^2} \\
&= \sqrt{1+1+16} = \sqrt{18} \\
&= 3\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
BC &= \sqrt{(-4+1)^2+(9-6)^2+(6-6)^2} \\
&= \sqrt{(-3)^2+(3)^2+(0)^2} \\
&= \sqrt{9+9} = \sqrt{18} \\
&= 3\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
CA &= \sqrt{(0+4)^2+(7-9)^2+(10-6)^2} \\
&= \sqrt{(4)^2+(-2)^2+(4)^2} \\
&= \sqrt{16+4+16} \\
&= \sqrt{36} \\
&= 6
\end{aligned}
$$

$$\text{Now, } AB^2+BC^2 = \left(3\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2 = 18+18 = 36 = AC^2$$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(iii) Let (–1, 2, 1), (1, –2, 5), (4, –7, 8), and (2, –3, 4) be denoted by A, B, C, and D respectively.

$$AB = \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2}$$
$$= \sqrt{4+16+16}$$
$$= \sqrt{36}$$
$$= 6$$

$$BC = \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2}$$
$$= \sqrt{9+25+9} = \sqrt{43}$$

$$CD = \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2}$$
$$= \sqrt{4+16+16}$$
$$= \sqrt{36}$$
$$= 6$$

$$DA = \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2}$$
$$= \sqrt{9+25+9} = \sqrt{43}$$

Here, AB = CD = 6, BC = AD = $\sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

Question 4:

Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, −1).

Answer 4:

Let P (x, y, z) be the point that is equidistant from points A(1, 2, 3) and B(3, 2, −1).

Accordingly, PA = PB

$$\Rightarrow PA^2 = PB^2$$
$$\Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$

Thus, the required equation is $x - 2z = 0$.

Question 5:

Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (−4, 0, 0) is equal to 10.

Answer 5:

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (−4, 0, 0) respectively.

It is given that PA + PB = 10.

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$$
$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$$

On squaring both sides, we obtain

$$\Rightarrow (x-4)^2 + y^2 + z^2 = 100 - 20\sqrt{(x+4)^2 + y^2 + z^2} + (x+4)^2 + y^2 + z^2$$
$$\Rightarrow x^2 - 8x + 16 + y^2 + z^2 = 100 - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} + x^2 + 8x + 16 + y^2 + z^2$$
$$\Rightarrow 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} = 100 + 16x$$
$$\Rightarrow 5\sqrt{x^2 + 8x + 16 + y^2 + z^2} = (25 + 4x)$$

On squaring both sides again, we obtain

$25(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x$

$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.